Orbital perturbations of transiting planets: A possible method to measure stellar quadrupoles and to detect Earth-mass planets

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ABSTRACT

The recent discovery of a planetary transit in the star HD 209458, and the subsequent highly precise observation of the transit lightcurve with HST, is encouraging to search for any phenomena that might induce small changes in the lightcurve. Here we consider the effect of the quadrupole moment of the parent star and of a possible second planet perturbing the orbit of the transiting planet. Both of these cause a precession of the orbital plane and of the periastron of the planet, which result in a long term variation of the duration and the period of the transits. For a transiting planet at 0.05 AU, either a quadrupole moment similar to that of the Sun or the gravitational tug from an Earth-like planet on an orbit of semimajor axis ~ 0.2 AU and a relative inclination near the optimal 45° would cause a transit duration time derivative of ~ 1 second per year.

Subject headings: planetary systems - stars: rotation

1. Introduction

The first transiting extrasolar planet has been identified around the G0 star HD209458 (Charbonneau et al. 2000; Henry et al. 2000b). The planet has a mass $M_p = 0.62 M_{\rm Jup}$ and orbits the star with a period of 3.52 days with semimajor axis $a = 0.047$ AU. Subsequent observations of the transit with the STIS camera on board HST have provided highly precise photometry of the lightcurve during the transit (Brown et al. 2001), with about 300 flux measurements of an accuracy close to 10^{-4} magnitudes. The shape of the lightcurve has been used to determine the radius of the planet, $R_p = 1.35 R_{Jup}$.

As discussed by Brown et al. (2001), the determination of transit lightcurves of such high precision opens up many opportunities for detecting other phenomena that could induce small changes in the lightcurve, such as the presence of satellites or rings around the planet. As more transits by other planets are discovered and observed with high precision, the opportunities to observe these effects will rapidly increase. This paper considers the effect of perturbations of the planet's orbit due to the quadrupole moment of the star or to a second planet. These perturbations should cause a precession of the periastron of an eccentric orbit, and a precession of the line of nodes if the orbital plane is not coincident with the stellar equator, or the plane of the second planet. The precession of the periastron and the line of nodes should result in long-term variations of the period and the duration of the transits. We will compute these two effects and discuss their detectability with future space missions.

2. Precession of the orbital plane

We consider first the effect of the precession of the orbital plane in the simple case where the orbit is circular. The orbital angular momentum of the planet is $L_p = M_p\, n\, a^2$, where $n = (GM_s/a^3)^{1/2}$ is the mean motion (or orbital angular frequency), and M_s is the mass of the star. We consider two possible causes for the precession of the orbital plane: the quadrupole moment of the star, which for solid body rotation has angular momentum $L_s \simeq k^2 M_s R_s^2 \omega_s$ (where R_s is the stellar radius, ω_s the angular frequency of rotation, and $k^2 \simeq 0.1$ for main-sequence stars; e.g., Ford, Rasio, & Sills 1999), or a second planet, with angular momentum $L_2 = M_2\, n_2\, a_2^2$, where we will assume $a_2 > a$. The precession occurs relative to the plane perpendicular to the total angular momentum vector $\mathbf{L}_t = \mathbf{L}_p + \mathbf{L}_{(s,2)}$, which will be referred to as the mean plane hereafter. The subindex $(s, 2)$ means that we are considering either the stellar rotation or a second planet.

Let i be the inclination of the orbital plane relative to either the stellar equator or the plane of the second planet, and i_p and $i_{(s,2)}$ the inclinations of these two planes relative to the mean plane. Obviously, $i = i_p + i_{(s,2)}$, and $L_p \sin i_p = L_{(s,2)} \sin i_{(s,2)}$. We define the x-axis as the intersection of the mean plane and the plane perpendicular to the line of sight, and β as the angle between the mean plane and the line of sight. The line of nodes is the intersection of the orbital plane of the planet and the mean plane, and forms an angle Ω relative to the x-axis. The orbital precession consists of the rotation of the angle Ω, with a precession angular frequency $\dot{\Omega}$.

2.1. Precession induced by the stellar quadrupole moment

The potential of a star expanded up to its quadrupole moment is given by

$$\phi = -\frac{GM_s}{r} + \frac{J_2}{2}\frac{GM_sR_s^2}{r^3}\left(3\sin^2\theta - 1\right) , \tag{1}$$

where R_s is the equatorial radius of the star, r is the distance to the center, θ is the angle relative to the equatorial plane, and J_2 is the quadrupole moment. Estimates of the quadrupole moment of the Sun are quite uncertain and over the range 10^{-7} to 10^{-6} (Godier & Rozelot 1999; Rozelot, Godier, & Lefebvre 2001). For self-similar main-sequence stars, $J_2 \propto \omega_s^2 R_s^3/M_s$. In the case of the star HD209458, the spectroscopically measured rotational velocity indicates a rotation slightly faster than the Sun (Queloz et al. 2000). We will use a fiducial value $J_2 = 10^{-6}$ in this paper.

To evaluate the time-averaged torque acting on the planet's orbit as a secular perturbation, we calculate the potential of a ring of mass M_p in the quadrupole potential of the star,

$$V = \frac{GM_sM_pR_s^2J_2}{2a^3}\int_0^{2\pi}\frac{d\varphi}{2\pi}(3\sin^2\varphi\sin^2 i - 1) = \frac{GM_sM_pR_s^2J_2}{2a^3}\left(\frac{3}{2}\sin^2 i - 1\right) . \tag{2}$$

The torque is

$$\tau = -dV/di = -\frac{3GM_sM_pR_s^2J_2}{4a^3}\sin 2i . \tag{3}$$

Since the component of the planet's angular momentum that is changing is $L_p \sin i_p$, the precession frequency is $\dot{\Omega} = \tau/(L_p \sin i_p)$. We will see below that the quantity determining the change in the transit width is $\dot{\Omega}\sin i_p$, which is

$$\dot{\Omega}\sin i_p = \frac{\tau}{L_p} = n\frac{R_s^2}{a^2}\frac{3J_2}{4}\sin 2i . \tag{4}$$

2.2. Precession induced by a second planet

The presence of a second planet will in general cause perturbations on all the orbital elements. When averaged over long timescales, the perturbations are classified as secular or resonant, and can be calculated with Lagrange's planetary equations (e.g., Murray & Dermott 1999, §6). Here we will consider only the secular precession of the line of nodes, in the most simple case when both the perturbing and perturbed planets are on circular orbits, which is easily obtained by replacing the planets by uniform rings of mass. Moreover, we will use the approximation $a_2 \gg a$. The case $a_2 < a$ is less interesting, because the most

accurate measurements of transits will be for planets very close to their stars; however, the precession effects caused by a planet interior to the transiting one can be calculated similarly to the exterior case.

The potential of a ring of radius a_2 and mass M_2, as a function of the cylindrical radius r on the plane of the orbit and the height z above the plane, is, up to second order in r/a_2 and z/a_2,

$$\phi = \frac{-GM_2}{a_2}\left(1 + \frac{r^2}{4a_2^2} - \frac{z^2}{2a_2^2}\right) . \tag{5}$$

The potential energy of interaction of this ring with another ring due to the inner planet, of mass M_p and radius a, and an inclination angle i between the two orbital planes, is

$$V = \frac{-GM_pM_2}{a_2}\left(1 + \frac{a^2}{4a_2^2} - \frac{3a^2}{8a_2^2}\sin^2 i\right) , \tag{6}$$

and the torque is

$$\tau = -dV/di = -\frac{3GM_pM_2a^2}{8a_2^3}\sin 2i . \tag{7}$$

Just as before, the precession frequency is

$$\dot{\Omega}\sin i_p = \frac{\tau}{L_p} = n\frac{M_2}{M_s}\frac{a^3}{a_2^3}\frac{3}{8}\sin 2i . \tag{8}$$

2.3. Effect on the transit duration

The properties of the transit lightcurve are determined by the angle α between the orbital plane of the planet and the line of sight. Simple spherical trigonometry shows that

$$\sin\alpha = \sin i_p \cos\beta\cos\Omega - \cos i_p \sin\beta , \tag{9}$$

where β is the angle between the mean plane and the line of sight. The duration of the transit is

$$t_d = \frac{2(R_s + R_p)}{n\,a}\cos\gamma , \tag{10}$$

where the angle γ is related to the impact parameter b of the transit by $(R_s + R_p)\sin\gamma = b = a\sin\alpha$. The relevant question to determine the observability of the precession of the orbital plane is if the rate of change of t_d can be measured over a reasonable observing time baseline $\Delta t_{obs} \sim 10$ years. We will see that the typical value of the precession period is much longer than 10 years, so only the time derivative dt_d/dt matters:

$$\frac{dt_d}{dt} = t_d\frac{a}{R_s + R_p}\frac{\sin\gamma}{\cos^2\gamma}\frac{d\alpha}{dt} = t_d\frac{a}{R_s + R_p}\frac{\sin\gamma}{\cos^2\gamma}\dot{\Omega}\sin i_p\cos\beta\sin\Omega . \tag{11}$$

Using $n\,t_d = 2(R_s + R_p)\cos\gamma/a$, we can reexpress this equation as

$$\frac{dt_d}{dt} = \frac{\dot{\Omega}\sin i_p}{n}\, 2\tan\gamma\,\cos\beta\,\sin\Omega\ . \tag{12}$$

We can now estimate the value of $\dot{\Omega}\sin i_p$ and see if the variation of the transit duration would be observable. A quadrupole moment of the star $J_2 \sim 10^{-6}$, with $a \sim 10R_s$, and $\sin 2i \sim 0.1$, yields a precession frequency $\dot{\Omega}\sin i_p \sim 10^{-9}n$, from equation (4). In the case of the perturbation by a planet, if its mass is $M_2 \sim 10^{-3}M_s$ (i.e., a gas giant), the effect of the planet would be comparable to that of the quadrupole for an orbital size $a_2 \simeq 30a$. For a 51-Peg type perturbed planet with $a \simeq 0.05$ AU, the second Jupiter-mass planet would be at $a_2 \simeq 1.5$ AU. Such a massive planet should also be detected from the Doppler measurements in any case. A more interesting case is if there is a lower mass planet (not detectable by Doppler measurements) on a smaller orbit. For example, for an Earth-like planet, with $M_2 \sim 3\times10^{-6}M_s$, at $a_2 \sim 5a \simeq 0.25$ AU, and $\sin 2i \sim 0.1$, the precession frequency is also $\dot{\Omega}\sin i_p \sim 10^{-9}n$. Therefore, from equation (12) (assuming that the trigonometric factors are of order unity), in these cases we should expect a time variation of the transit duration $dt_d/dt \sim 10^{-9}$.

What is the highest accuracy to which we can measure dt_d/dt ? In the STIS observations of Brown et al. (2001) of HD209458, the flux varies by $\sim 1\%$ during a time $t_c \simeq 0.01$ days at the beginning and the end of the transit. Each one of their flux measurements is obtained from a 60 s integration and has a relative accuracy of $\sim 10^{-4}$. Therefore, each data point taken during the falling or rising part of the lightcurve gives us the starting or ending time of the transit to an accuracy $\sim (10^{-4}/0.01)t_c$, or ~ 10 seconds. Brown et al. have measured nearly 100 data points in the rapidly varying part of the lightcurve, so their present data should allow to determine the transit duration to an accuracy approaching 1 second. Repeating the measurement 3 years later would yield dt_d/dt to an accuracy of 10^{-8}.

The star HD209458 has magnitude $V = 7.64$, implying that the number of photons received by HST over a 60 s integration time is $\sim 10^9$. Therefore, the photometric accuracy achieved by Brown et al. could be improved by only a factor ~ 3 by reaching the photon shot noise limit; alternatively, a telescope aperture of 0.7 m could reach the same photometric accuracy with improvements in the detection efficiency. A dedicated mission observing all the ~ 1000 transits over a period of 10 years of stars similar to HD209458 with the same accuracy as Brown et al. would yield dt_d/dt to an accuracy $\sim 10^{-9.5}$.

We can easily change the parameters of the examples discussed previously to consider cases where the orbital perturbations would be detectable with the more easily achievable accuracy of $dt_d/dt \simeq 10^{-8}$. For example, a perturbing planet with $M_p = 3\times10^{-5}M_s$,

$a_2/a = 4$, and $\sin 2i = 0.3$, would be detectable at the 5-σ level.

3. Precession of the periastron

The orbits of the 51-Peg type planets tend to be circular, as expected owing to tidal dissipation in the planet, which circularizes the orbit on a timescale (Goldreich & Soter 1966)

$$\tau_e = \frac{4}{63} Q \, n^{-1} \frac{M_p}{M_s} \left(\frac{a}{R_p} \right)^5 , \tag{13}$$

where the factor Q is inversely proportional to the dissipation rate. For the planet Jupiter, $Q \sim 10^5$ has been found observationally from the tidal effect on Jupiter's satellites (Ioannou & Lindzen 1993). Assuming the same Q for HD209458, for which case $M_p/M_s \simeq 5 \times 10^{-4}$ and $a/R_p \simeq 75$, we find $\tau_e \sim 10^7$ years. How the parameter Q may vary among different planets is highly uncertain, because the origin of the dissipation is not well understood (e.g., Murray & Dermott 1999, §4.13). So, while internal dissipation in the planets is a plausible explanation for the orbital circularity of the closest planets, it is possible that some of the closest planets may have eccentric orbits. In addition, the extreme sensitivity of the time τ_e to a/R_p implies that planets that are only slighly further out from their star can have significant eccentricities even if Q is constant. Orbital eccentricities can also be excited by planetary companions (e.g., Rivera & Lissauer 2000). In fact, among known extrasolar planets, high eccentricities are common when $a > 0.1$ AU (Butler et al. 2000), and the planet around HD217107 has $a = 0.072$ AU and $e = 0.14 \pm 0.05$ (Fischer et al. 1999). We can expect that transits of planets in eccentric orbits will be discovered in the near future.

We note here that tidal dissipation in the star can also circularize the orbit, and make the orbit coplanar with the stellar equator. However, the timescale for dissipation in the star is much longer than the age of the system, and is in any case similar to the timescale for orbital decay (Rasio et al. 1996; Zahn 1977).

3.1. Precession rates

The periastron of an eccentric orbit should precess due to three effects: the relativistic precession, and the same two effects considered previously, the stellar quadrupole moment and the perturbations from other planets.

The relativistic precession rate is given by (e.g., Landau & Lifshitz 1951)

$$\dot{\varpi} = n \frac{3}{1 - e^2} \left(\frac{n \, a}{c} \right)^2 , \tag{14}$$

where e is the eccentricity, and we denote the precession of the periastron as $\dot{\varpi}$. For an orbital period of ten days, $a \sim 0.1$ AU, and small eccentricity, this gives $\dot{\varpi} \simeq 4 \times 10^{-7} n \simeq 10^{-4}\,\mathrm{yr}^{-1}$.

For the stellar quadrupole moment, we assume here that the orbital plane coincides with the stellar equator. The potential of the star is given by equation (1) with $\theta = 0$. The precession rate for small eccentricities is easily obtained using the epicycle approximation. The orbital angular frequency is $n^2 = (1/r) d\phi/dr$, and the epicycle frequency is (e.g., Binney & Tremaine 1987)

$$\kappa^2 = r\frac{dn^2}{dr} + 4n^2 = n^2 - \frac{3GM_s R_s^2 J_2}{r^5} \ . \tag{15}$$

The precession frequency is

$$\dot{\varpi} = n - \kappa = \frac{3J_2 R_s^2}{2a^2} n \ , \tag{16}$$

where we have substituted $a = r$ and have used $\dot{\varpi} \ll n$. Typically, $J_2 \simeq 10^{-6}$ and $R_s \lesssim 0.1a$, so $\dot{\varpi} \lesssim 10^{-8} n$. Except in rapidly rotating stars, the effect of the stellar quadrupole is always much less than the relativistic precession.

For the effect from a second planet, repetition of the same method with the potential in equation (5) at z=0 yields a precession frequency

$$\dot{\varpi} = \frac{3M_2 a^3}{4M_s a_2^3} n \ . \tag{17}$$

For an Earth-like planet with $M_2/M_s \simeq 3 \times 10^{-6}$, and $a_2 \simeq 2a$, the precession rate is $\dot{\varpi} \simeq 3 \times 10^{-7} n$, comparable to that from the relativistic precession. All the three effects we have discussed cause an advance of the periastron, and therefore the total precession is the sum of the three effects.

3.2. Detectability of the periastron precession

The periastron precession causes both the period and the duration of the transits to change. We discuss first the variation of the period.

In the epicycle approximation (valid for small eccentricities), the eccentric orbit of the planet can be described by the motion along an epicycle relative to a circular orbit, with coordinates $x(t)$ along the inward radial direction and $y(t)$ in the backward tangential direction along the orbit. For an orbit in a quasi-Keplerian potential (where the precession of the periastron is small), we have (Binney & Tremaine 1987):

$$x(t) = ae\cos(\kappa t + \psi_0) \ ; \tag{18}$$

$$y(t) = -2ae\sin(\kappa t + \psi_0)\ . \tag{19}$$

The central time of the transits is determined by $y(t)$ at the time of the transit. Between two successive transits, the epicycle phase $\psi = \kappa t + \psi_0$ changes by $\Delta\psi = 2\pi(\dot{\varpi}/n)$, and therefore $y(t)$ changes by $\Delta y = 2ae\,2\pi(\dot{\varpi}/n)\,\cos(\kappa t + \psi_0)$. Thus, the observed transit period, P_t, is (to first order in e)

$$P_t = \frac{2\pi}{n}\left(1 - \frac{\Delta y}{2\pi a}\right) = \frac{2\pi}{n}\left(1 - 2e\frac{\dot{\varpi}}{n}\cos\psi\right)\ . \tag{20}$$

A first possible method to detect the periastron precession is to compare the transit period P_t with the true orbital period $P = 2\pi/n$ determined from the Doppler measurements. The accuracy of the Doppler measurements are at present $\epsilon_D \sim 0.1$ times the velocity variation amplitude, and they are unlikely to improve very much due to limitations set by photospheric turbulence. Hence, the orbital period can be determined to an accuracy $\Delta P/P \simeq \epsilon_D P/(2\pi N^{1/2} t_0)$, where N is the number of Doppler measurements and t_0 is the total time of observation. For $\epsilon_D = 0.1$, $N = 1000$ and $P/t_0 = 10^{-3}$, the accuracy achieved is $\Delta P/P \sim 10^{-6}$, allowing detection only for a planet with $M_2/M_s \gtrsim 10^{-4}$ if $e \sim 0.2$ and $a_2/a \simeq 2$.

A second possibility is to detect the change of P_t with time:

$$\frac{dP_t}{dt} = 4\pi\, e\left(\frac{\dot{\varpi}}{n}\right)^2 \sin\psi\ . \tag{21}$$

If the time of each transit can be measured to an accuracy of 1 second, the accuracy of the period after observing $N = 1000$ transits would be $1\mathrm{s}/N^{3/2} \sim 10^{-4.5}$ s, and for an observing time of 10 years the accuracy of dP_t/dt would reach $\sim 10^{-13}$. For $e \sim 0.1$, this allows for a 10-σ detection of a planet with $M_2/M_s = 10^{-5}$ and $a_2 = 2a$.

Next we evaluate the change in the duration of the transit, which is caused by the variation of the planet's velocity during transit as the periastron precesses. From equation (19), the velocity of the planet at transit is $v = na(1 - 2e\cos\psi)$ (notice that for the purpose of computing this velocity we can use the approximation $\kappa \simeq n$), so the duration is

$$t_d = \frac{2(R_s + R_p)\cos\gamma}{v} \simeq 2\,\frac{(R_s + R_p)}{na}\cos\gamma\,(1 + 2e\cos\psi)\ , \tag{22}$$

and its time derivative is

$$\frac{dt_d}{dt} = 4e\,\frac{\dot{\varpi}}{n}\frac{(R_s + R_p)}{a}\cos\gamma\,\sin\psi\ . \tag{23}$$

As mentioned previously, the perturbation by an Earth-like planet with $a_2 = 2a$ implies a periastron precession rate $\dot{\varpi}/n \sim 3\times 10^{-7}$; for $R_s/a \sim 0.05$, $4e \sim 1$, we infer $dt_d/dt \sim 10^{-8}$,

which is not difficult to detect as discussed in §2.3. From equations (8) and (12), we see that the transit duration change due to the precession of the orbital plane would be comparable if $\sin 2i \simeq 0.1$.

In principle, the change in transit duration due to the precession of the periastron and the lines of nodes can be distinguished by the effect on the shape of the lightcurve. Precession of the periastron changes only the velocity, so the shape of the lightcurve does not change. However, precession of the line of nodes changes the impact parameter of the transit. Of course, separating the two effects requires a higher photometric accuracy, and will increase the measurement errors unless the effect of the periastron precession is also measured from the change in the transit period. The most favorable case would be when the planet transits over the edge of the star, when the lightcurve shape has the fastest change with a variation of the impact parameter.

It is simple to see which one of the two measurements, the change in the period or the duration of the transit, can yield the highest accuracy to measure the ratio $\dot{\varpi}/n$. The period derivative is always measured to an accuracy better than the transit duration derivative by a factor P/t_0, where t_0 is the time of observation. Equating this to the ratio of equations (21) and (23), we find that the transit duration change yields a higher accuracy until the time of observation is $t_0 < \dot{\varpi}^{-1}\, 2(R_s + R_p)\cos\gamma/a$, and the period change gives the most accurate determination after that. For a close-in planet with $2R_s/a \simeq 0.1$, and an Earth-mass perturbing planet with $a_2/a \simeq 2$, we find $\dot{\varpi}^{-1} \sim 3 \times 10^4$ years. Therefore, the transit duration change should yield the periastron precession rate with the highest accuracy, except for substantially more massive perturbing planets.

4. Conclusions

Accurate photometry of transit lightcurves of extrasolar planets allows the detection of slow orbital perturbations which affect the lightcurve. Here, we have considered the perturbation due to a stellar quadrupole moment or to a second planet as possible causes for the precession of the line of nodes and of the periastron of the orbit. Both types of precession cause a time variation of the duration of the transit, and the periastron precession also causes a variation of the period. The rate of this variation is given by equations (12), (23), and (21).

The quadrupole moment of the star is important for the close-in planets, at $a \sim 10R_s$. These planets are likely to be on circular orbits and therefore the precession of the line of nodes should usually be the most important effect. From observations of the stellar spectrum

one can infer the projected rotational velocity of the star, and monitoring the CaII flux can also reveal the rotation period (Henry et al. 2000a). In addition, spectroscopic variations of the star during the transits can in principle be used to measure the angle Ω of the line of nodes (Queloz et al. 2000). If all these quantities are measured, it should be possible to infer from the precession rate of the orbital plane, the quadrupole moment of the star J_2, which is sensitive to the state of rotation of the inner parts of the star.

The precession induced by other planets will be added to that caused by the stellar quadrupole, and only the sum of both effects is observable. However, once the stellar rotation is measured the effect of the quadrupole is predictable to some extent, and it should be negligible compared to the effect of another planet in many cases (for large a or M_2).

The precession induced by a second planet depends only on the quantities $(M_2/M_s)(a/a_2)^3$, and $\sin 2i$ (see eqs. [8] and [17]). In principle, by measuring both the orbital and periastron precession from the change in the duration and the shape of the lightcurve, or from measuring the change in the period as well, both of these quantities could be determined assuming that there is only one perturbing planet. Even then, the mass and semimajor axis of the perturbing planet cannot be separately inferred, and the detected precession could be the result of a low-mass planet close to the transiting planet, or a more massive planet further away, or a disk of planetessimals.

In the examples mentioned previously in this paper, we have considered perturbations by planets of low enough mass that they would not be detected via the Doppler measurements of the star. However, transits could also allow the detection of perturbations by other known planets. Several cases of systems with two or three planets have now been discovered, some of which seem to be on resonant orbits (Marcy et al. 2001). Measuring the orbital and periastron precessions in one of these systems, if they were to show transits, could allow the determination of their relative orbital inclination. For a case with $M_2/M_s \sim 10^{-3}$, $a_2/a = 2^{2/3}$, the precession periods may be as short as hundreds of years, making it possible that a whole series of transits may be observed over a reasonable time of ~ 10 years as the line of nodes precesses, and enabling very accurate measurements of the orbital perturbations. A more general analysis than that presented here would be required to compute the precession rates when the two planets are not coplanar, are both on eccentric orbits, and are possibly resonant, including also the effect of changes of the eccentricity on the transit period and duration.

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